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COMPLEX LOADING AND CERTAIN PROBLEMS OF THE BIRFURCATION OF THE ELASTIC-PLASTIC PROCESS

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A differential-nonlinear version of the theory of plasticity based on the model. of a linear anisotropically hardening plane-plastic medium [7] and the isotropy postulate [8], which has been verified by experimant under complex loading [4-6], has been developed in [1-3] for plane deformation paths. In particular, dependences have been established between the stress and strain increments at an angular point of the loading trajectory.

The dependences mentioned are extended below to the case of arbitrary orientation of a small additional loading relative to the preceding plane trajectory. The result obtained permits an approach to the formulation and solution of problems about the influence of the loading history on the magnitude of the critical parameters corresponding to bifurcation points of the process (Shanley formulation) and the bifurcation of states (Karman formulation) of thin walled structural elements from the aspect of the differential-nonlinear version of the theory of plasticity. Certain qualitative aspects of this question are clarified in the model of a plate subjected to bilateral compression [9]. In particular, it is shown that a birfurcation point of the process precedes the bifurcation point of the state if the method of fastening and loading the model during the branching imposes no constraints on the angle of the deformation trajectory broken-line. Within the framework of the differential-linear versions of the theory of plasticity, this result was obtained earlier for a three-dimensional body [10-12]. A deduction is made that the Shanley critical load, determined taking the active loading history into account, exceeds the critical load determined by the Hencky-Nadai theory of deformations without taking account of the loading history.

1. ON THE EQUATIONS OF STATE IN A SMALL NEIGHBORHOOD OF AN ANGULAR POINT OF THE LOADING TRAJECTORY. Let the loading of a linear anisotropically hardening plane-plastic medium [7] be characterized by an arbitrary trajectory OAB (Fig. 1) in the S_1S_3 plane of the five-dimensional Il'iushin [8] space of stresses. We express the stress $\mathbf{S}$ and strain $\mathbf{E}$ vector components in terms of the tensor components $\sigma_{ij}, \varepsilon_{ij}$ $(i, j \sim x, y, z)$ by using the equalities.

$$S_1 = \sqrt{1/2}(\sigma_x - \sigma_y), \quad S_2 = \sqrt{3/2}(\sigma_z - \sigma), \quad S_3 = \sqrt{2}\,\tau_{xy},$$

$$S_4 = \sqrt{2}\,\tau_{yz}$$

$$S_5 = \sqrt{2}\,\tau_{zx} \quad (\sigma_{ii} = \sigma_i;\ \sigma_{ij} = \tau_{ij},\ i \neq j;\ \sigma = 1/3(\sigma_x + \sigma_y + \sigma_z))$$

$$E_1 = \sqrt{1/2}(\varepsilon_x - \varepsilon_y), \quad E_2 = \sqrt{3/2}(\varepsilon_z - \varepsilon), \quad E_3 = \sqrt{1/2}\,\gamma_{xy},$$

$$E_4 = \sqrt{1/2}\,\gamma_{yz}$$

$$E_5 = \sqrt{1/2}\,\gamma_{zx} \quad (\varepsilon_{ii} = \varepsilon_i;\ 2\varepsilon_{ij} = \gamma_{ij},\ i \neq j;\ \varepsilon = 1/3(\varepsilon_x + \varepsilon_y + \varepsilon_z))$$

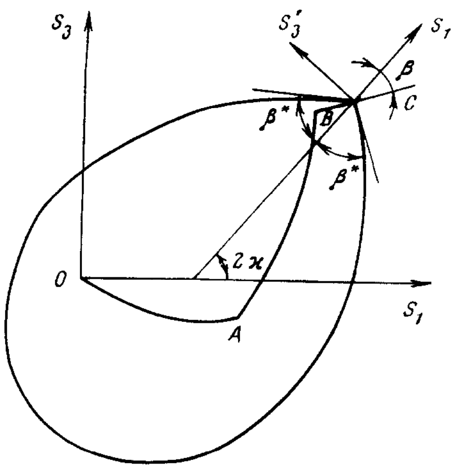

Fig. 1

Let us apply a small additional load $\mathbf{dS} = \mathbf{BC}$ from the point B and let us write the expression for the strain increment $\mathbf{dE}$. We still consider the vector to be located in the plane S_1S_3.

If the response of a plane medium to strain anisotropy is characterized by a logarithmic function of the hardening $F(\omega) = k \ln (c / \omega)$ then according to [3] we can write in the auxiliary $CS_1'S_3'$ coordinate system (such a representation of the relation $\mathbf{dS} \sim \mathbf{dE}$ is used for the first time)

$$dS_1' = G_p (2\beta_{13}^*) \, dE_1', \quad dS_3' = G_g (2\beta_{13}^*) \, dE_3' \tag{1.1}$$

$$G_p = \left(\frac{1}{2G} + B_{11}\right)^{-1}, \quad G_g = \left(\frac{1}{2G} + B_{12}\right)^{-1}$$

$$B_{11} = \frac{1}{2k}\left[\frac{J_0(2\alpha)}{\ln (2C/\alpha)} - 2\alpha J_1(2\alpha)\right] J_0(2\alpha) + B_{12} \tag{1.2}$$

$$B_{12} = \frac{\alpha^2}{k}[J_0^2(2\alpha) + J_1^2(2\alpha)], \quad 2\alpha = \frac{\pi}{2} - \beta_{13}^*$$

Here G is the elastic shear modulus, $J_0(2\alpha)$, $J_1(2\alpha)$ are Bessel functions, $2\beta_{13}^*$ is the angle formed by the tangents to the loading line Σ_{13} at the loading point C (Fig. 1). The functions G_p and G_g, called the moduli of continuing loading and additional loading henceforth, are independent of the form of the trajectory, and proportional loading, in particular, can be used to determine them.

It follows from (1.1) and (1.2) that the relation between $\mathbf{dS}$ and $\mathbf{dE}$ for arbitrary trajectories is determined completely by the angle of the singularity $2\beta_{13}^*$ on the loading line Σ_{13} and its orientation $2\varkappa$ in the plane S_1S_3. The argument $2\beta_{13}^*$ of the functions G_p, G_g and the orientation parameter $2\varkappa$ depends essentially on the loading history and are functionals of the process.

It should be noted that the invariance of the dependence of the angle of the singularity at the loading point and the vectors of the increments $\mathbf{dS}$ $\mathbf{dE}$ obtained above within the framework of the model [7] was formulated earlier by Kliushnikov as a postulate in the version of the theory of plasticity which he proposed [13].

The method of analytical construction of the loading line Σ_{13} has been elucidated in [14] for the model of a linear anistropically-hardening plane-plastic medium, based on the concept of slip. In particular, the value of the angle $2\beta_{13}^*$ and its orientation $2\varkappa$ are determined by using two functions $2\alpha = \alpha_2(t) + \alpha_1(t)$ and $2\varkappa = \alpha_2(t) - \alpha_1(t)$, where $\alpha_1(t)$ and $\alpha_2(t)$ are the boundaries of a set of slip directions, and the trajectory arclength is taken as the time parameter t. At an arbitrary loading time, α_1 and α_2 are calculated from the condition that the resistance to shear S_m and the velocity of its increment $\partial S_m / \partial t$ in the direction of m of developing slip are, respectively, equal to the tangential stress τ_m acting over the slip area, and the velocity of the increment in the tangential stress $\partial \tau_m / \partial t$

Appropriate algorithms have been proposed in [3, 15, 16]; here we limit ourselves just to the following remarks.

Let the trajectory OAB be smooth at B or accompanied by a break as shown in Fig. 1, but let the angle β between the direction dS and the bisectrix of the angle $2\beta_{13}^*$ at the point B not exceed the quantity β_0 determined by the formula

$$\operatorname{tg} \beta_0 = \frac{1}{2\alpha_B \ln (2c / \alpha_B)} - \frac{J_1(2\alpha_B)}{J_0(2\alpha_B)}, \quad \alpha_B = \alpha(t_B) \tag{1.3}$$

In this case the small additional load dS from the end of the trajectory OAB does not result in freezing of the slip systems occurring at the time $t_B - 0$ preceding the breakpoint of the trajectory at the point B and

$$\alpha_{1,2}(t_B + 0) = \alpha_{1,2}(t_B - 0), \quad 2\alpha_B = \alpha_2(t_B \pm 0) + \alpha_1(t_B \pm 0)$$

Thus the value, of the angles $2\beta_{13}^*$ corresponding to the points B and C of the trajectory agree within the limit $dS \to 0$, and the relations (1.1), (1.2) determine the differential-linear relation $dS_i' \sim dE_i'$ independent of the direction dS within the cone $\beta \leqslant \beta_0$ of the total additional load.

If $\beta \geqslant \pi - (\beta_{13}^*)_B$, then unloading occurs according to the elastic law and $G_p = G_g = 2G$. For $(\beta_0 < \beta < \pi - (\beta_{13}^*)_B)$ the increment dS will result in partial or total freezing of the slip systems, and therefore, to a jump change in the angles $2\beta_{13}^* = \pi - 2(\alpha_1 + \alpha_2)$ and $2\varkappa = \alpha_2 - \alpha_1$. In particular, if the vector dS is directed along the line Σ_{13}, then $(2\beta_{13}^*)_B = \pi$ and the loading point on Σ_{13} becomes regular.

For an imcomplete additional load $(\beta_0 < \beta < \pi - (\beta_{13}^*)_B)$ the relation between dS_i' and dE_i' determined by (1.1) and (1.2) depends essentially on the direction dS and therefore, is differentially nonlinear. The angle of the singularity $2\beta_{13}^*$ at the point C and its orientation $2\varkappa_C$ are hence determined by (1.4), where the parameter α_C is computed from (1.5)

$$\begin{aligned} (2\beta_{13}^*)_C &= (2\beta_{13}^*)_B - 4(\alpha_B - \alpha_C) \\ 2\varkappa_C &= 2\varkappa_B + 2(\alpha_B - \alpha_C) \end{aligned} \tag{1.4}$$

$$\operatorname{tg}(\beta - 2\alpha_B - 2\alpha_C) = \frac{1}{2\alpha_C \ln (2C / \alpha_C)} - \frac{J_1(2\alpha_C)}{J_0(2\alpha_C)} \tag{1.5}$$

The results obtained for plane-plastic deformation on the basis of the isotropy postulate remain valid for arbitrary plane and spatial monotonic loading paths [3] arbitrarily located in the five-dimensional Il'iushin space [8]. Conditions for the model of a plane medium to satisfy the isotropy postulate are mentioned in [4]. Replacing the logarithmic strengthening function $F(\omega)$ by its general possible form [17] does not alter the crux of the question.

Now let us consider the case when the additional load dS is in the plane Π passing through the S_1' axis but rotated arbitrarily with respect to the preceding plane trajectory OAB (Fig. 1).

Proceeding from the above-mentioned invariance of the dependences of the loading angle $2\beta^*$ and the vector increments $\mathbf{dS}$, $\mathbf{dE}$, as well as independences of the functions G_p, G_g from the form of the trajectory for any plane and spatial monotonic loading paths, it is natural to assume that the mentioned invariance and independence of G_p and G_g hold for arbitrary loading processes. Hence, by analogy with (1.1) for the small additional load $\mathbf{dS}$ oriented arbitrarily with respect to the preceding trajectory, we can write

$$dS_1' = G_p(2\beta_{1\Pi}^*)\, dE_1', \quad dS_\Pi' = G_g(2\beta_{1\Pi}^*)\, dE_\Pi' \tag{1.6}$$

Here $2\beta_{1\Pi}^*$ is the angle formed by the tangents to the line of intersection between the loading surface Σ and the plane Π.

Let the additional load $\mathbf{dS}$ not result in freezing of the slip systems in the first and second cases. Then the increment dE_1' is determined just by the component dS_1' and is independent of the components dS_3' and dS_Π', in particular $dS_3' = dS_\Pi' = 0$ can be assumed without changing the quantity dE_1'. Hence, for an arbitrary point B of the trajectory we obtain on the basis of (1.1) and (1.6)

$$G_p(2\beta_{13}^*) = G_p(2\beta_{1\Pi}^*) \tag{1.7}$$

According to (1.2), here G_p is a single-valued increasing function and total additional loading can be realized from an arbitrary point of the loading trajectory, hence (1.7) holds upon compliance with the condition

$$\beta_{13}^* = \beta_{1\Pi}^* \tag{1.8}$$

(The independence of G_p and G_g from the angle β within the total additional loading cone does not mean constancy of these functions with the change in β_{13}^*.)

Taking into account that the orientation of Π relative to the $S_1'S_3'$ plane was selected arbitrarily, we arrive at the deduction that for any strain process the solid angle $2\beta^*$ formed by the tangents to the loading surface Σ at the loading point, is a hypercone of rotation. This result permits extension of (1.1) to the case of arbitrary orientation of the additional load vector relative to the plane trajectory OAB (Fig. 1). We have

$$dS_1' = G_p(2\beta^*)\, dE_1', \quad dS_i' = G_g(2\beta^*)\, dE_i', \quad i = 2, 3, 4, 5 \tag{1.9}$$

Here, as above, the continuing loading G_p and additional load G_g moduli are determined by the relationships (1.2) $S_1'S_2'S_3'S_4'S_5'$ are a mutually orthogonal local coordinate system with origin at the running point C of the loading trajectory, where, S_1' is the axis of the hypercone of rotation formed by tangents to the surface Σ at the loading point

2. ON BIFURCATION OF THE PROCESS AND STATE OF AN IDEALIZED MODEL OF A PLATE. Let us consider the Kliushnikov model consisting of two parallel square plates $2a \times 2a$ with a total cross-sectional area f separated by a spacing $2h$ and deformable in their planes by two pairs of rigid levers (see Fig. 1 in [9]. The ends of the levers and the forces P_x, P_y acting on them remain in the xy plane; the process of the change in force during the time t is arbitrary. The plane equilibrium mode becomes unstable and model buckling occurs upon the achievement of the combinations $P_x = f\sigma_x$, $P_y = f\sigma_y$ of certain critical values P_x^*, P_y^* dependent on the loading history.

In an infinitesimal neighborhood of the branch point, the equilibrium conditions of the external forces and moments, as well as the relation between the strain increment and the deflection dw of the center of the model are given by the equations [9]

$$dQ_x + dQ_y = 0 \tag{2.1}$$
$$P_i dw + L dQ_i = hf/2\,(d\sigma_i^+ - d\sigma_i^-);$$
$$dw = aL/2h\,(d\varepsilon_i^+ - d\varepsilon_i^-) \quad (i \sim x, y;\ a \ll L)$$

where dQ_i is the reaction of the support during buckling and $d\sigma_i\pm$, $d\varepsilon_i\pm$ are the additional stresses and strains to the initial state. (Here and henceforth, quantities referring to the upper plate of the model are marked with the plus superscript, while those referring to the lower plate are marked with the minus. We consider the compressive stress and strain positive; we do not exclude from consideration the unloading of the lower plate during buckling.

Going over to the variables

$$S_1 = \sqrt{1/2}\,(\sigma_x - \sigma_y), \quad S_2 = \sqrt{1/6}\,(\sigma_x + \sigma_y)$$
$$E_1 = \sqrt{1/2}\,(\varepsilon_x - \varepsilon_y), \quad E_2 = \sqrt{1/6}\,(\varepsilon_x + \varepsilon_y), \quad S_i = E_i = 0, \quad i \geqslant 3$$

we obtain from (2.1), after eliminating dQ_x and dQ_y

$$dE_1^+ = dE_1^-, \quad \frac{aL}{h^2} S_2 (dE_2^+ - dE_2^-) = \sqrt{\frac{2}{3}}\,(dS_2^+ - dS_2^-)$$

These latter relationships can be rewritten thus in projections on the axis of the auxiliary $S_1'S_2'$ coordinate system (Fig. 1):

$$dE_1'^+ \cos 2\varkappa^+ - dE_1'^- \cos 2\varkappa^- = dE_2'^+ \sin 2\varkappa^+ - dE_2'^- \sin 2\varkappa^- \tag{2.2}$$

$$\frac{aL}{h^2} S_2 (dE_1'^+ \sin 2\varkappa^+ - dE_1'^- \sin 2\varkappa^- + dE_2'^+ \cos 2\varkappa^+ - dE_2'^- \cos 2\varkappa^-) =$$
$$\sqrt{\frac{2}{3}}\,(dS_1'^+ \sin 2\varkappa^+ - dS_1'^- \sin 2\varkappa^- + dS_2'^+ \cos 2\varkappa^+ - dS_2'^- \cos 2\varkappa^-)$$

First, let us consider the problem of determing the critical load in the Shanley formulation. Let the deformation process in an infinitesimal neighborhood of the branch point occur without freezing of the slip system in both the upper and lower plates of the model because of the additional load increment dP_i ($P_i dP_i > 0$). It will later be shown that such an assumption results in a minimum critical load. In the total additional load case under consideration we obtain

$$(2\beta^*)^+ = (2\beta^*)^-, \quad 2\varkappa^+ = 2\varkappa^-;$$

and we have on the basis of (1.9)

$$dS_1'\pm = G_p\,(2\beta^*)\,dE_1'\pm, \quad dS_2' = G_g\,(2\beta^*)\,dE_2'\pm$$

Taking account of the last dependences, we obtain from the second equation in (2.2) for the Shanley critical load ($S_2 = S_2^*$)

$$S_2{}^* = \sqrt{\frac{2}{3}}\,\frac{h^2}{aL}\,[G_p(2\beta^*)\sin^2 2\varkappa + G_g(2\beta^*)\cos^2 2\varkappa] \qquad (2.3)$$

Partial or total freezing of the slip results in an increase in the moduli G_p and G_g according to (1.2) and (1.4), hence (2.3) obtained under the assumption of total additional loading at the initial instant of buckling of the plate model, determines the minimal critical load.

Now, let us examine the problem of determining the critical Karman load $S_2 = S_2{}^{**}$. In this case $dP_i = 0$ and the vectors of the additional load increments $\mathbf{dS}^\pm$ in the upper and lower plates of the model are equal in magnitude but opposite in sign

$$\mathbf{dS}^- = -\mathbf{dS}^+, \quad \beta^- = \pi + \beta^+$$

Without imposing preliminary constraints here on the magnitude of the angles of the additional load directions β^+ and β^-, we can write for the upper and lower plates of the model from (1.9)

$$dS_1{}'^\pm = G_p{}^\pm dE_1{}'^\pm, \quad dS_2{}'^\pm = G_g\, dE_2{}'^\pm$$

We hence obtain from the first equation of the system (2.2)

$$\left(\frac{\sin 2\varkappa^+}{G_g{}^+} + \frac{\sin 2\varkappa^-}{G_g{}^-}\right)\operatorname{tg}\beta^+ - \left(\frac{\cos 2\varkappa^+}{G_p{}^+} + \frac{\cos 2\varkappa^-}{G_p{}^-}\right) = 0 \qquad (2.4)$$

Taking into account that for a given subcritical state the moduli $G_p{}^\pm$, $G_g{}^\pm$ and the parameters $2\varkappa^\pm$ are the functions of the angle β^+, we conclude that the last equation is the definition of the additional load direction $\mathbf{dS}^+$ ($\mathbf{dS}^- = -\mathbf{dS}^+$) for bifurcation of the Karman state.

It can be shown that the solution $\beta^+ = \beta_k{}^+$ of (2.4) is given by

$$\operatorname{tg}\beta_k{}^+ = \left(1 + \frac{2G}{G_p}\right)\left(1 + \frac{2G}{G_g}\right)^{-1} \operatorname{ctg} 2\varkappa \qquad (2.5)$$

if $\beta_k{}^+$ does not exceed the values of β_0 and β^*. Here G_p, G_g and $2\varkappa$ are evaluated at the time $t_B - 0$ preceding bifurcation of the state. Bifurcation is hence accompanied by a complete additional loading of the upper and unloading by an elastic law of the lower plates of the model. Therefore, we can write

$$\begin{aligned} dS_1{}'^+ &= G_p dE_1{}'^+, \quad dS_2{}'^+ = G_g dE_2{}'^+ \\ dS_2{}'^- &= 2G dE_2{}'^-, \quad dS_2{}'^- = 2G dE_2{}'^- \end{aligned} \qquad (2.6)$$

If the dependences (2.6) are substituted into the second equation in (2.2), then we obtain for the Karman critical load

$$S_2{}^{**} = \sqrt{\frac{2}{3}}\,\frac{h^2}{aL}\,(D_p \sin^2 2\varkappa + D_g \cos^2 2\varkappa) \qquad (2.7)$$

$$D_p = 2G_p\left(1 + \frac{G_p}{2G}\right)^{-1}, \quad D_g = 2G_g\left(1 + \frac{G_g}{2G}\right)^{-1}$$

Since $D_p > G_p$, $D_g > G_g$, a simple comparison of (2.3) with (2.7) results in the condition

$$S_2^* < S_2^{**}$$

The equality $S_2^* = S_2^{**}$ is possible only within elasticity limits when $G_p = G_g = 2G$, and we have for the critical load $(S_2 = S_{2e})$

$$S_{2e} = 2\sqrt{\frac{2}{3}\frac{Gh^2}{aL}}$$

As the magnitude of the plastic deformation grows in the subcritical state, it can turn out that the solution β_k^+ of (2.4) will not satisfy the condition $\beta_k^* < \beta^*$ and β_0. (This can occur in the case $\beta^* < \beta_0$.). Then the vectors dS^+ and dS^- simultaneously, or only dS^- will emerge from the zone of total additional loading and unloading, respectively, and will enter the zone of incomplete additional loading. In this case it is impossible to represent the formula for S_2^{**} in a compact explicit form, however, as numerical computations show, even here the condition $S_2^* < S_2^{**}$ remains valid. It has therefore been shown that bifurcation of the process of a plate model precedes bifurcation of the state. As noted above, this deduction is an extension of the known result on [10-12] to the case of a differential-nonlinear variant of plasticity theory based on the slip concept.

3. ON THE DEPENDENCE OF THE SHANLEY CRITICAL LOAD ON THE LOADING HISTORY. The papers [18-20], particularly, are devoted to a study of this question. Some aspects of the influence of the loading history on the critical parameters are examined below on the basis of (2.3) written for arbitrary loading paths for a plate model.

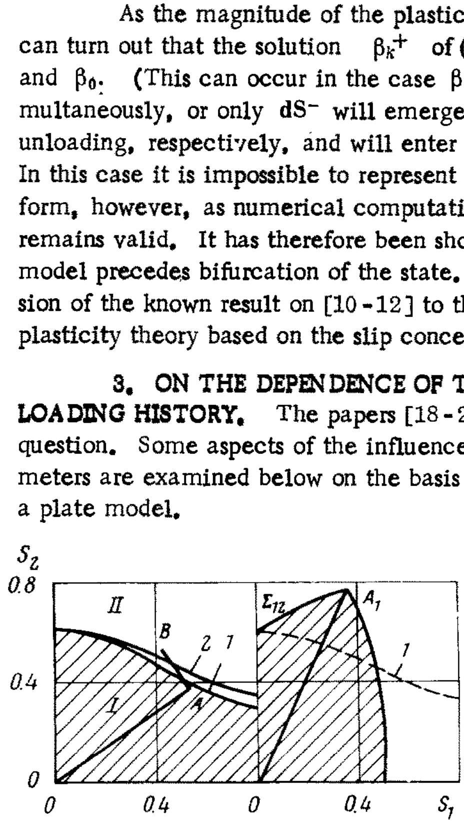

Fig. 2

If the strain process is monotonic in the subcritical state [1.], in particular, is proportional, then the differential-nonlinear version of the theory of plasticity which was used will degenerate into the Hencky-Nadai theory of deformation. As has been shown in [19], in this case there exists an absolute stability domain I (Fig. 2). If it is taken into account in addition that $G_p = {}^2/_3\, E_t$, $G_g = {}^2/_3\, E_s$, for monotonic strain, where E_s and E_t are the secant and tangent moduli of the $\sigma_u \sim \varepsilon_u$ diagram under simple loading, then on the basis of (2.3) the equation of the line 1 bounding the domain I can be represented in the form

$$\frac{S^*}{S_{2e}} = \frac{E_s}{2G}\left(\frac{1}{\sin 2\varkappa} - n\sin 2\varkappa\right) \quad \left(S^* = \frac{S_2^*}{\sin 2\varkappa}, \quad n = 1 - \frac{E_t}{E_s}\right)$$

The line mentioned is constructed in Fig. 2 for the aluminum alloy AK-6 ($G = 0.27 \cdot 10^6$ кг / см^2, $\tau_s = 410$ кг / см^2, $c = 0{,}105\,\pi$, $F = k \ln (c / \omega)$, $G / k = 8.5$). The magnitude of the critical load S_{2e} is taken as one in the computations and $S_2^* / S_{2e} = 0.6$. for $2\varkappa = \pi / 2$. The zone $S_{2e} > 1$ is the zone of elastic instability.

Violation of the monotonicity condition in the subcritical state, just as at the time of bifurcation, results in an increase in the moduli G_p, G_g and to growth of the critical load as a whole. In this connection *1* is the absolute stability domain even for nonmonotonic active loading paths. A change in the quantity S_2^* under loading by a two-link trajectory OAB with a $\pi / 2$ breakpoint angle is shown in Fig. 2 by line *2*. The difference between curves 1 and 2 is insignificant. The authors of [21] arrive at an analogous deduction by an experimental investigation of the influence of the loading history on the magnitude of the critical parameters of a cylindrical shell loaded by an axial force and internal pressure according to two-lined trajectories. Let us note that the principal axes of the stress tensor under complex loading retain their constant direction in the cases under consideration. There is reason to assume that the influence of the history on the critical load of thin-walled structure elements can be substantial under complex loading with rotation of the principle axes of the stress tensor.

As follows from (2.3), for a model a significant rise in S_2^* is achieved on loading paths close to complete freezing of the slip system, when G_p, $G_g \to 2G$. The following possibility also merits attention. The domain within the initial Σ_{12}° or the next Σ_{12} loading line (Fig. 2) satisfying the condition $S_2 < S_{2e}$ is the elastic stability domain. Therefore, a significant rise in S_2^* can be obtained (points of zone II are reached) by preliminary loading of the model in the plastic domain outside the bifurcation limits, which is realizable upon the imposition of additional kinematic constraints. Unloading and repeated loading within the line Σ_{12} with the additional constraints removed does not result in bifurcation of equilibrium if $S_2^* < S_{2e}$. In particular, there is a discussion of an analogous possibility in [20].

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